


Practical Application of Geotechnical Limit Analysis in Limit State Design

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Limit State Design

- Ultimate Limit State Design to Eurocode 7
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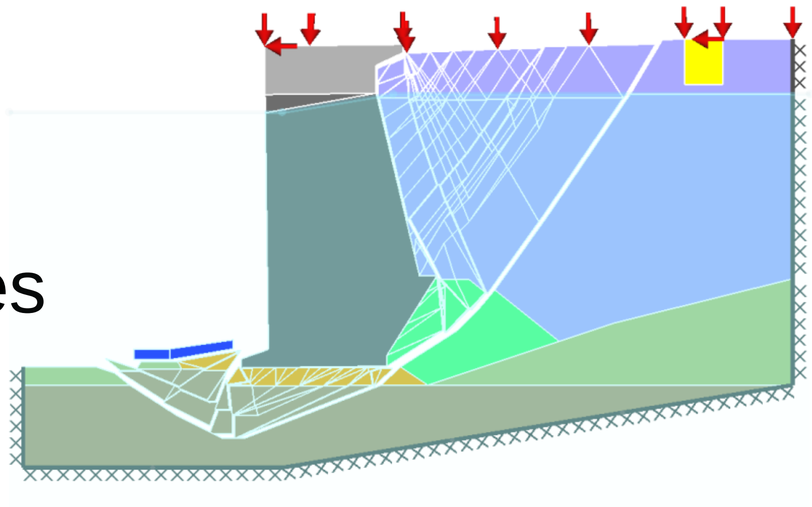


Specific Objectives

- At the end of this lecture , delegates should be aware of:
  - the core principles underlying Limit State Design
  - the implementation of Limit State Design as exemplified by Eurocode 7
  - the use of Limit State Design with computational limit analysis
  - challenges and advantages of Limit State Design

Limit state analysis / design

What is 'limit state' analysis / design?

- Limit states are *states beyond which the structure no longer satisfies the relevant design criteria*
  - focus on what might go wrong
  - e.g. ultimate limit state (**ULS)** and serviceability limit state (**SLS)**

- Not directly related to specific analysis methods

What is ‘limit state’ analysis / design? [2]*

- ‘An understanding of limit state design can be obtained by contrasting it with *working state design*:
  - Working state design: Analyse the expected, working state, then apply margins of safety
  - Limit state design: Analyse the unexpected states at which the structure has reached an unacceptable limit
  - Make sure the limit states are unrealistic (or at least unlikely)’

*Brian Simpson, Arup

The ultimate 'limit state'

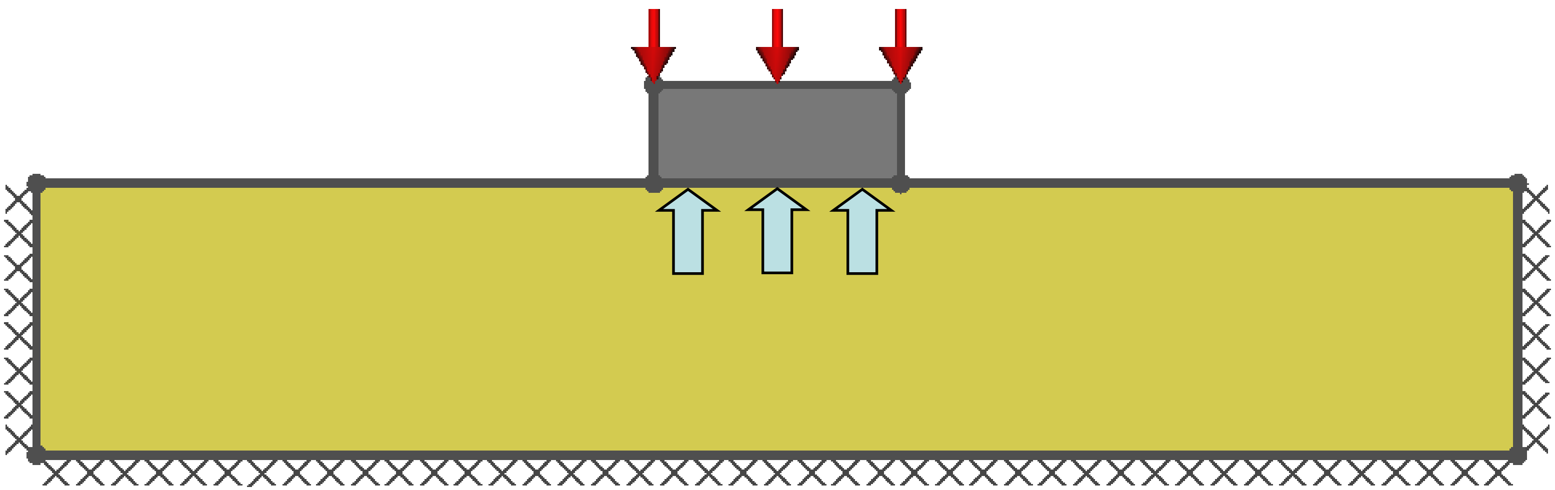
- According to EC 0 ('Basis of structural design'):

(4)P The following ultimate limit states shall be verified where they are relevant :

– loss of equilibrium of the structure or any part of it, considered as a rigid body ;
– failure by excessive deformation, transformation of the structure or any part of it into a mechanism, rupture, loss of stability of the structure or any part of it, including supports and foundations ;
– failure caused by fatigue or other time-dependent effects.

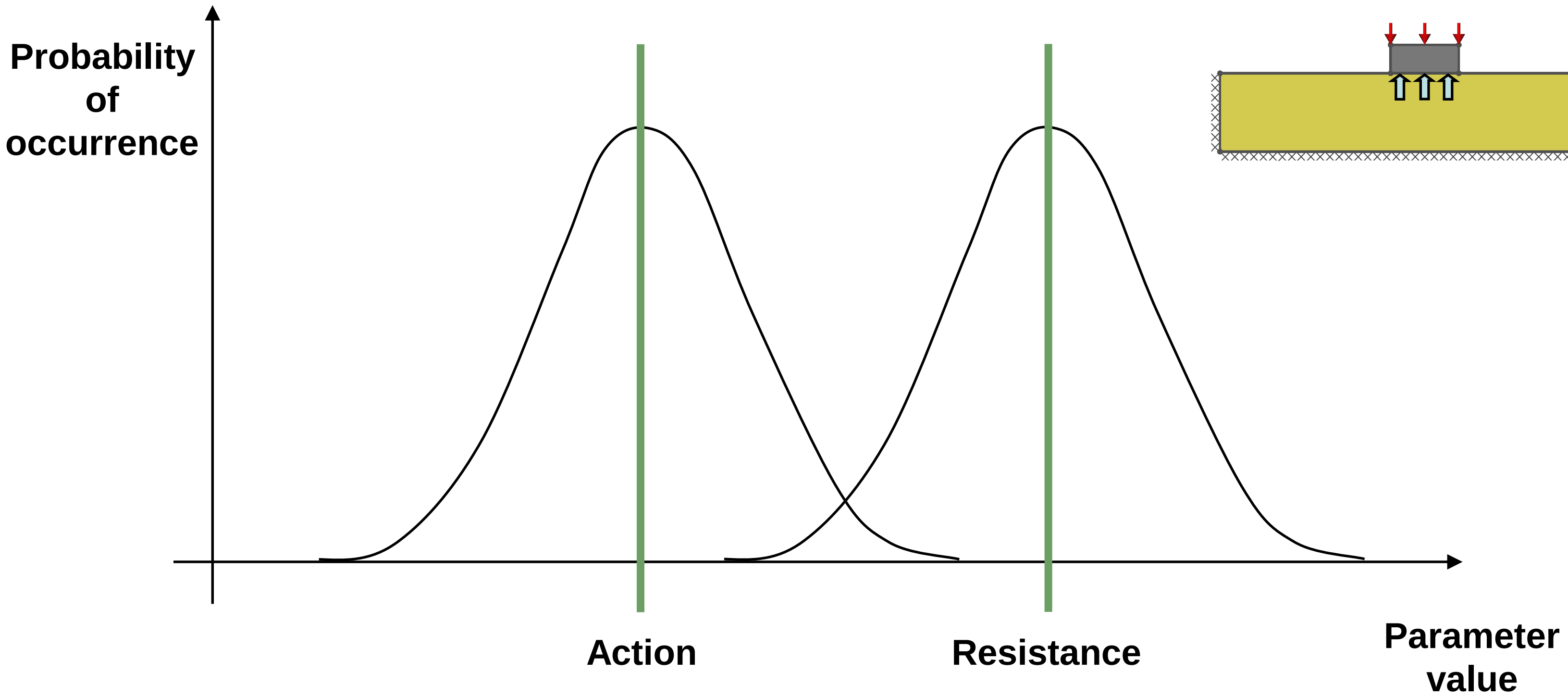
[Serious consequences of failure mean this must be rendered a remote possibility]

Implementation in practice



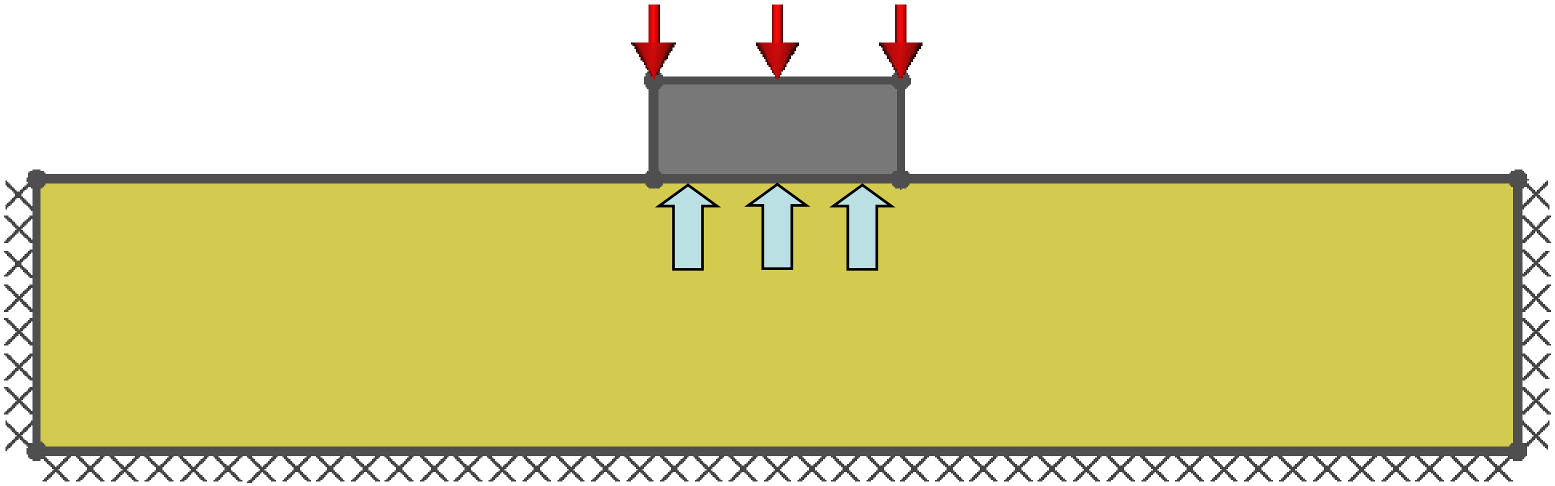
Consider actions (loads) and resistances (i.e. resistance available at collapse)

Implementation in practice [2]



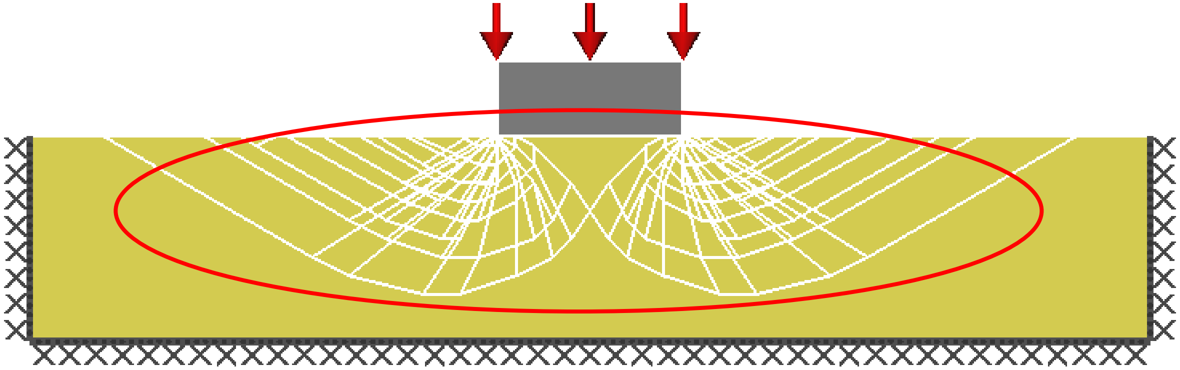
Partial factors are applied to Actions and Resistances

But…



Should we take the resistance at the footing/soil interface?

But… [2]



Or in the soil?

Implementation of Limit State Design in Eurocode 7

Eurocode 7

- Eurocode 7 has been under development since 1980.
- Use of Eurocode 7 will be mandatory in the UK in 2010.
  - Original draft had 1 Design Approach (DA)
  - Now has 3 different Design Approaches. Individual countries are free to adopt one or more approaches for national use

Eurocode 7

Eurocode 7 moves away from:

- working stresses
- overall factors of safety

to:

- limit state design approach
- partial factors
- unified approach

Eurocode 7

It introduces a range of separate checks:

- Ultimate Limit States (ULS)

  EQU : loss of equilibrium
  STR : failure of the structure
  GEO : failure of the ground
  UPL : failure by uplift
  HYD : hydraulic heave

- Serviceability Limit States (SLS)

Existing codes

Existing codes often combine **SLS** and **ULS**:

Advantages:

- Simple
- Only one calculation required

Disadvantages:

- Not necessarily transparent,
- ‘Safety’ factors typically only applicable for a specific subset of parameters

Eurocode 7

In the UK the checks against failure in the ground (GEO) and in the structure (STR) must be checked using ‘Design Approach 1 (DA1)’ which requires that two checks are performed:

- Design combination 1 (DA1/1)
- Design combination 2 (DA1/2)

Loads (Actions)

Different factors are applied if loads (Actions) are:

- Permanent
- Variable
- Accidental

Different factors are applied if loads (Actions) are:

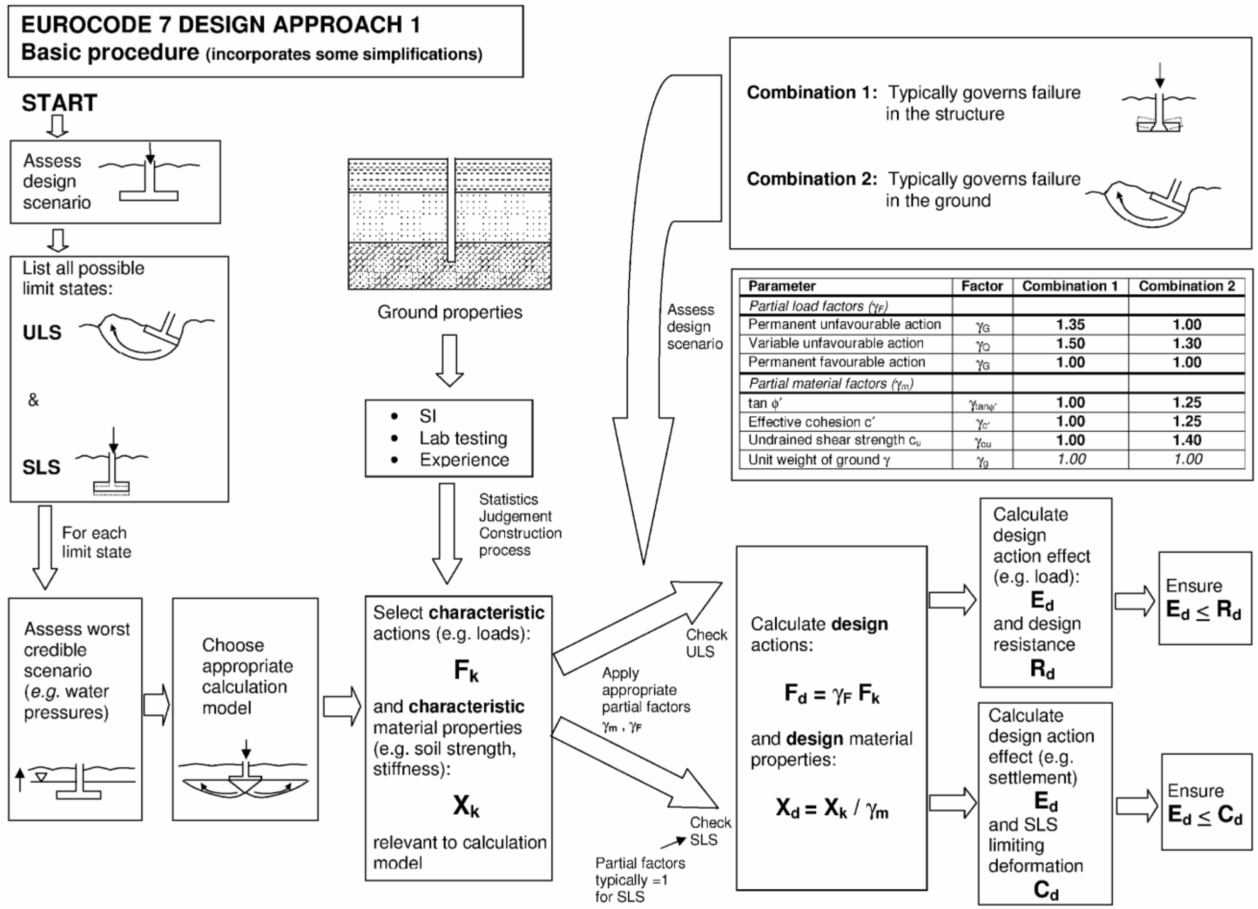
- Favourable
- Unfavourable

i.e assist or resist collapse (this may not be clear at outset)

Partial Factors

	EC7 DA1/1	EC7 DA1/2	Conventional bearing capacity	BS8002 (Retaining walls)
Permanent unfavourable load	1.35	1.0	1.0	1.0
Variable unfavourable load	1.5	1.3	1.0	1.0
Permanent favourable load	1.0	1.0	1.0	1.0
c'	1.0	1.25	1.0	1.2
$\tan\phi'$	1.0	1.25	1.0	1.2
c_u	1.0	1.4	1.0	1.5
Resistance	1.0	1.0	2.5 – 3.5	N/A

Quick Guide



Parameter	Factor	Combination 1	Combination 2
Partial load factors (γ_F)			
Permanent unfavourable action	γ_G	**1.35**	**1.00**
Variable unfavourable action	γ_Q	**1.50**	**1.30**
Permanent favourable action	γ_G	**1.00**	**1.00**
Partial material factors (γ_m)			
tan ϕ'	$\gamma_{\tan\phi'}$	**1.00**	**1.25**
Effective cohesion c'	$\gamma_{c'}$	**1.00**	**1.25**
Undrained shear strength c_u	γ_{cu}	**1.00**	**1.40**
Unit weight of ground γ	γ_g	*1.00*	*1.00*

Eurocode 7 Example: Foundation Design

Example – foundation design

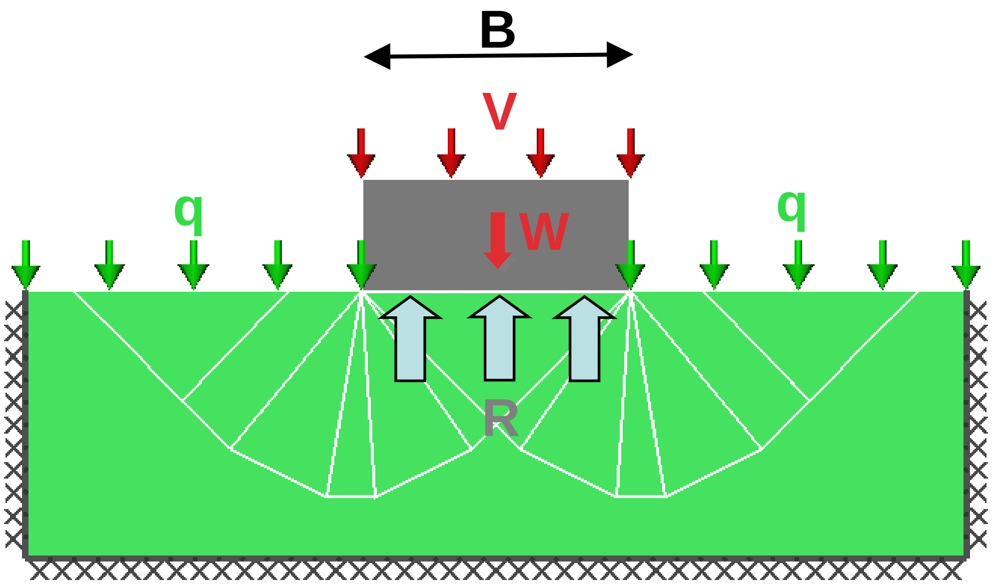
Simple design case:

- Footing on undrained (clay soil)

Examine:

- Conventional design approach
- Design to Eurocode 7

Example – foundation design [2]



c_u = 70 kN/m^2

γ = 20 kN/m^3

q = 10 kN/m^2 *

W = 40 kN/m

B = 2m

*Assume q is a permanent load

Terzaghi's bearing capacity equation:

$$R/B = cN_c + qN_q + \tfrac{1}{2}\gamma BN_\gamma$$

For the undrained case:

$$R/B = (2+\pi)c_u + q$$

Example - foundation design [3]

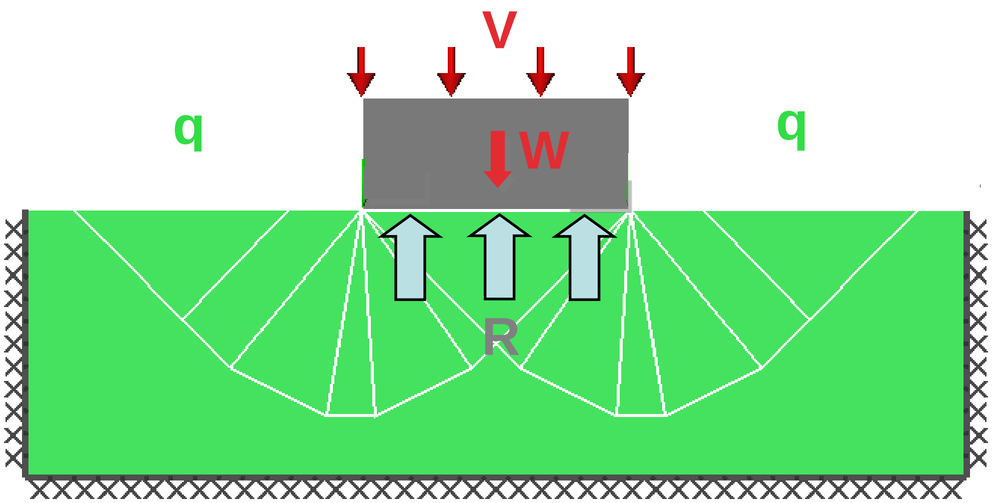
Design to Eurocode 7

Eurocode 7 requires a change in conceptual approach. Instead of finding a collapse load and factoring it, it is necessary to define **Actions** and **Resistances** in the problem and ensure that

Actions ≤ Resistances

To ensure reliability of the design, **Partial factors** are applied to the actions and resistances/material properties.

Example – foundation design [4]



$c_u = 70$ kN/m^2

$\gamma = 20$ kN/m^3

$q = 10$ kN/m^2

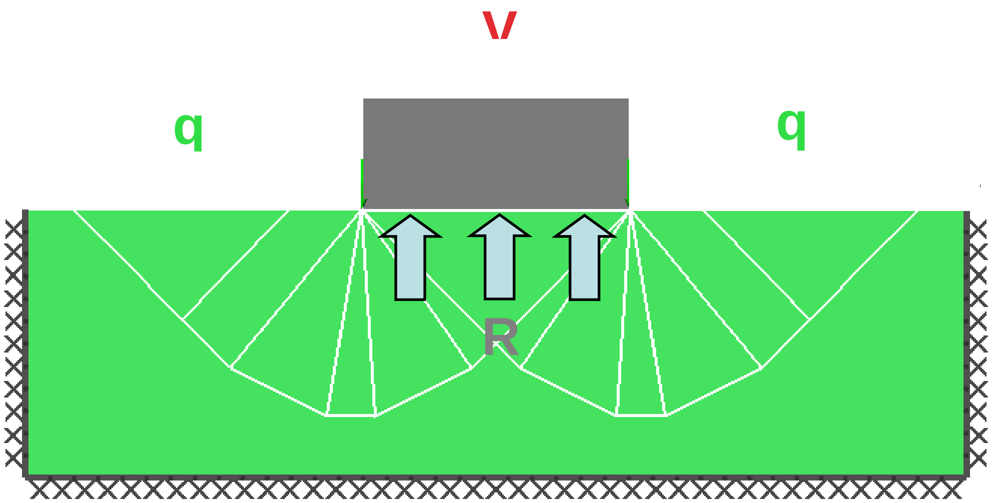
W = 40 kN/m

B = 2m

The actions (E) driving the footing to collapse are:

- Foundation load V
- Foundation self weight W

Example – foundation design [5]



$c_u = 70\ kN/m^2$

$\gamma = 20\ kN/m^3$

$q = 10\ kN/m^2$

$W = 40\ kN/m$

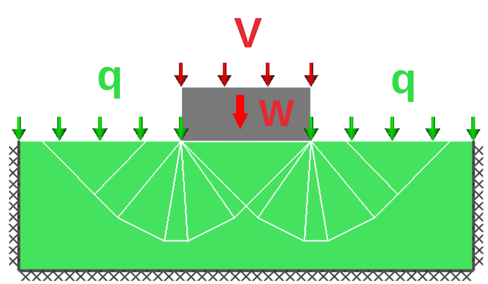
$B = 2m$

The Resistance R opposing collapse of the footing is provided by:

- Soil resistance R (applied at base of footing)

Example - foundation design [6]

	Conventional	DA1/1	DA1/2
Design Actions (E_d) $E=V+W$	V+40	1.35(V+40)	1.0(V+40)
Design Resistance (R_d) $R=B(5.14c_u+q)$	$\frac{2(5.14\times 70+10)}{3.0}$	$\frac{2(5.14\times {}^{70}\!/_{1.0}+10)}{1.0}$	$\frac{2(5.14\times {}^{70}\!/_{1.4}+10)}{1.0}$
R_d (kN/m)	247	740	534
$E_d<R_d$	V≤207 kN/m	V≤508 kN/m	V≤494 kN/m



c_u = 70 kN/m^2
γ = 20 kN/m^3
q = 10 kN/m^2
W = 40 kN/m
B = 2m

Eurocode 7

Comments:

- The most critical case must be taken in Eurocode 7. In this case maximum permissible load on the foundation is V = 494 kN/m (in Design Combination 2)
- This value is significantly higher than that allowed by the conventional design approach (V = 207 kN/m)
- Why?

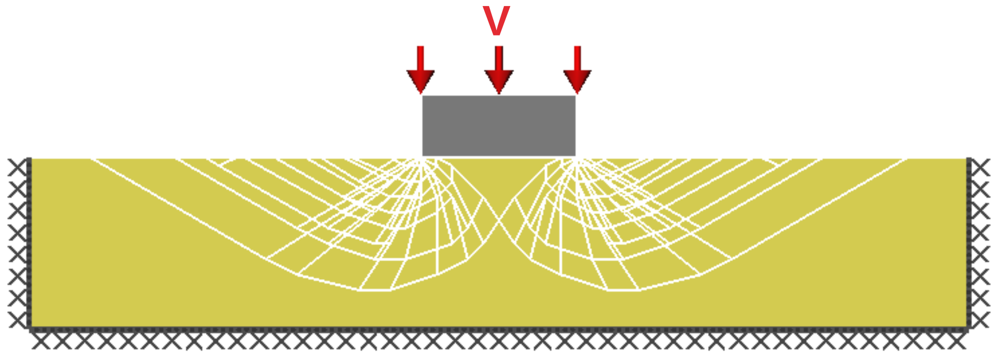
Eurocode 7 [2]

- Eurocode 7 presents a significant change in philosophy compared to most previous codes.
- The preceding calculations are reliability based and ensure that the probability of collapse at the ULS is very low.
- The calculation says nothing about the SLS which must be addressed by a separate calculation. The F.O.S = 3 method implicitly addressed both SLS and ULS.

Why two calculations?

One reason can be related to the non-linearity of some limit analysis calculations when checking geotechnical stability.

Consider the bearing capacity of a footing sitting on a cohesionless soil:



The exact solution is given by: $V/B = 0.5\gamma B N\gamma$

Where $N\gamma$ must be computed numerically.

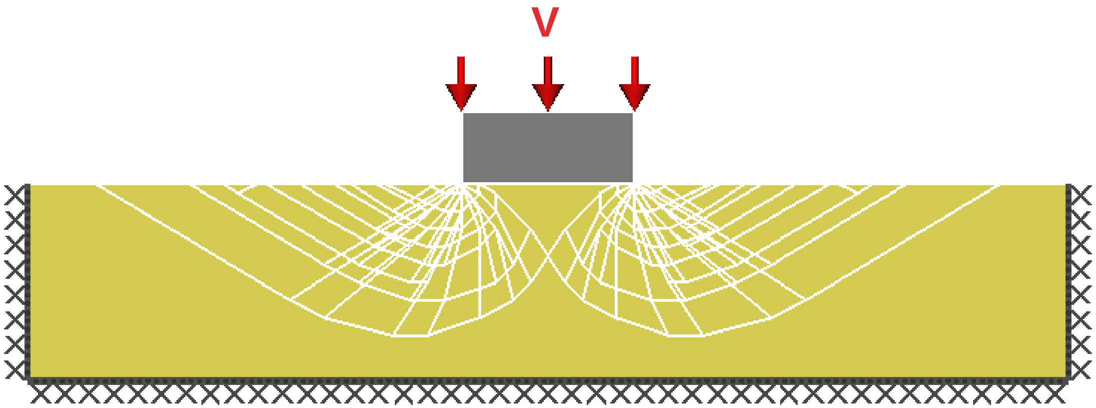
Why two calculations? [2]

Values of N_γ are highly sensitive to the value of ϕ:

ϕ	N_γ
25º	6.49
30º	14.8
35º	34.5
40º	85.6
45º	234
50º	743

Why two calculations? [3]

Example results (unfactored) for B = 2m, γ = 16 kN/m^3:

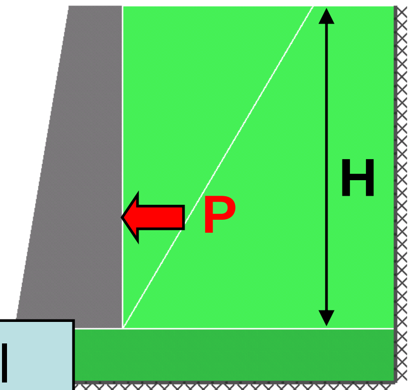


ϕ'	*V* (kN/m)	*V*/2750
40º	2750	1.0
38.4º	2030	1.35
34º	930	3.0

Why two calculations? [4]

Active soil thrust for a smooth wall P is given by:

$$P = \frac{1}{2} K_a \gamma H^2, \qquad K_a = \frac{1 - \sin\phi'}{1 + \sin\phi'}$$



DA1/2 does not become critical until ϕ'>45°. However if soil strength is relevant in other parts of the problem (e.g. base sliding) then DA1/2 typically dominates.

ϕ	[illegible]	[illegible]/2)	*ratio*
30°	[illegible]	[illegible]	1.23
35°	[illegible]	[illegible]	1.27
40°	0.22	0.28	1.31
45°	0.17	0.23	1.35
50°	0.13	0.18	1.38

Factors of Safety and overdesign factors

Factors of Safety

Many different definitions of factors of safety are used in geotechnical engineering. Three in common usage are listed below:

1. Factor on load.
2. Factor on material strength.
3. Factor defined as ratio of resisting forces (or moments) to disturbing forces (or moments).

The calculation process used to determine each of these factors for any given problem will in general result in a *different failure mechanism*, and a *different numerical factor*. Each FoS must therefore be interpreted according to its definition.

The Ultimate Limit State

- In general any given design is inherently stable and is nowhere near to its ultimate limit state.
- In order to undertake a ULS analysis it is necessary to drive the system to collapse by some means.
- This can be done implicitly or explicitly. In many conventional analyses the process is typically implicit. In a general numerical analysis it must be done explicitly.

Driving the system to ULS

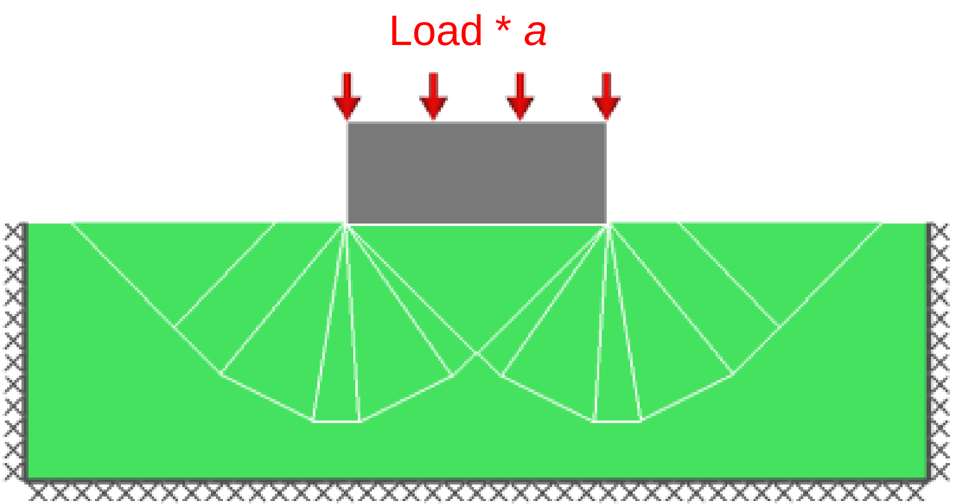
There are three general ways to drive a system to ULS corresponding to the three FoS definitions previously mentioned:

1. Increasing an existing load in the system.
2. Reducing the soil strength
3. Imposing an additional load in the system

Computational Limit Analysis (including LimitState:GEO) solves problems using Method 1. However it can be straightforwardly programmed to find any of the other two types of Factors of Safety.

Method 1

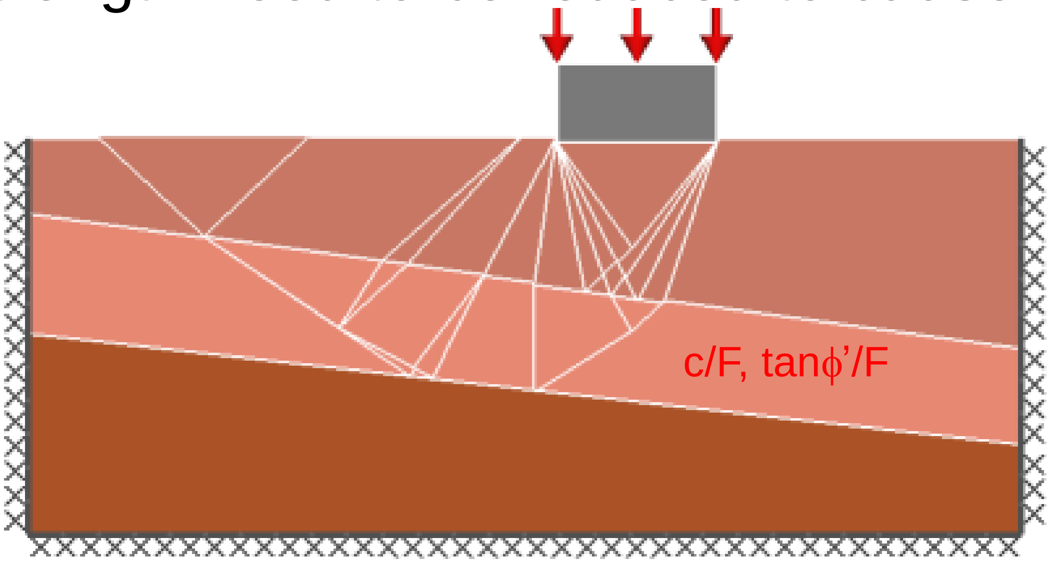
Question: how much bigger does the load need to be to cause collapse, or, by what factor *a* does the load need to be increased to cause collapse:



This factor *a* is the ‘adequacy’ factor.

Method 2

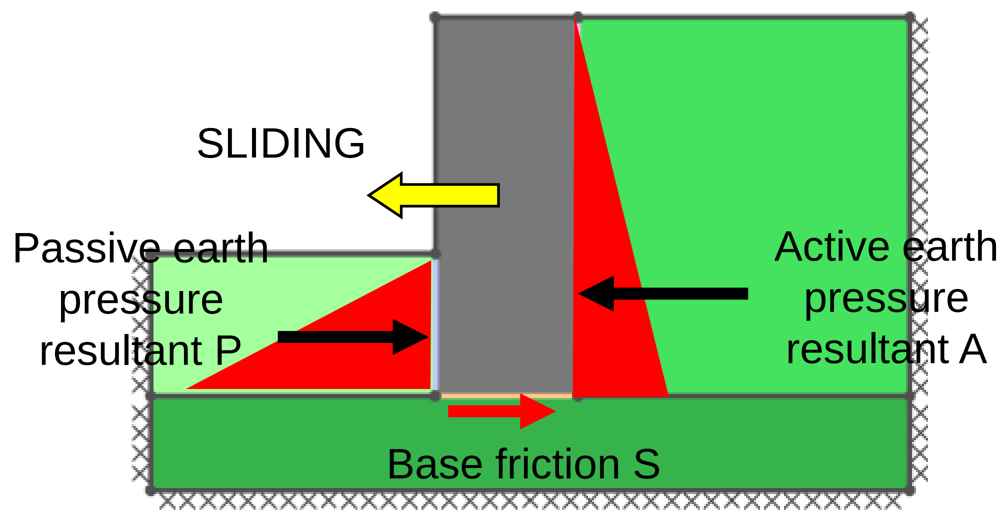
Question: how much weaker does the soil need to be under the design load to cause collapse, or, by what factor *F* does the soil strength need to be reduced to cause collapse:



This factor *F* is the factor of safety on strength.

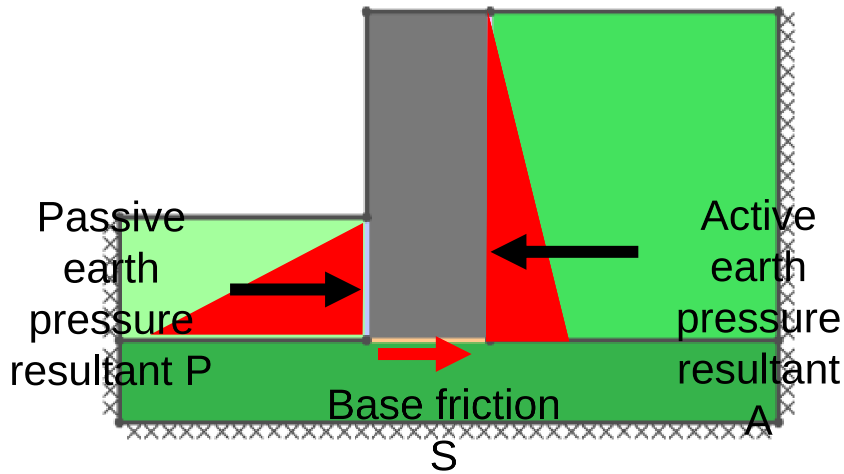
Method 3

Question: If the soil is failing around the structure, what is the ratio *R* of resisting forces to disturbing forces



This factor $R = A / (P + S)$ is a factor of safety.

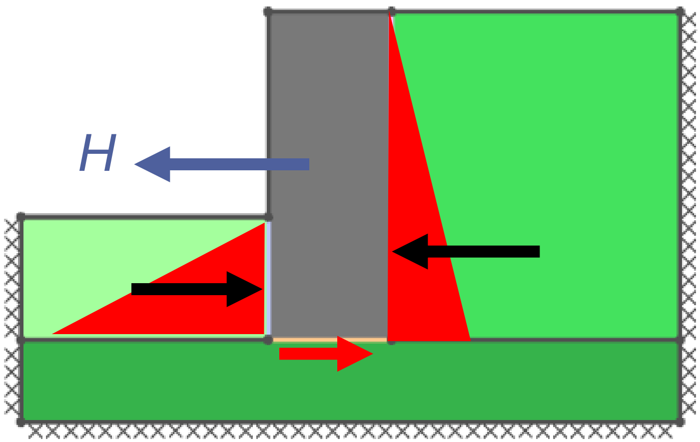
Method 3: analysis



Note that if $R > 1$:

- the passive earth pressure and base friction significantly exceed the active earth pressure
- the system is therefore completely out of equilibrium.
- These assumed earth pressures are not possible without some external disturbing agent.

Method 3: numerical analysis



- In a numerical analysis, equilibrium is required at all times.
- Therefore apply a 'hypothetical' external force *H* in the direction of assumed failure. Increase this force until failure occurs.
- Then determine ratio of other resisting to disturbing forces as before, but ignore *H* itself.
- Mode of failure must be *pre-determined* in this method.

Equivalence of Methods

- At failure, Method 1(a = 1) and Method 2 (F = 1) are identical

- Method 3 (R = 1) can be identical but only for the given failure mechanism

Eurocode 7: DA1, DA2 and DA3

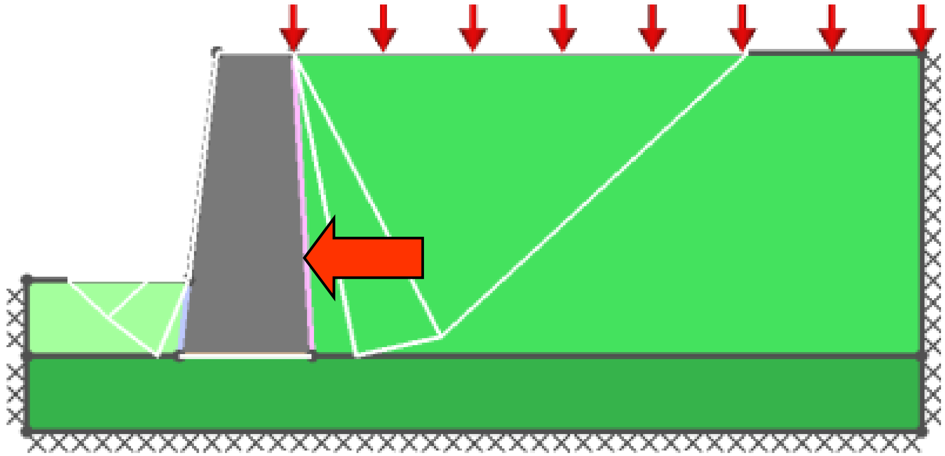
	EC7 DA1/1	EC7 DA1/2	EC7 DA2	EC7 DA3
Permanent unfavourable load	1.35	1.0	1.35	1.0/1.35
Variable unfavourable load	1.5	1.3	1.5	1.0/1.5
Permanent favourable load	1.0	1.0	1.0	1.0
c'	1.0	1.25	1.0	1.25
$\tan\phi'$	1.0	1.25	1.0	1.25
c_u	1.0	1.4	1.0	1.4
Resistance	1.0	1.0	1.1/1.4	N/A

Eurocode Methods

- DA1/1 and DA2 are Method 3 approaches (also Method 1 in simpler cases)
- Advantages:
  - Familiar approach to geotechnical engineers for straightforward problems
- Disadvantages:
  - Much more challenging to conceptualise for complex problems
  - More difficult to analyse numerically

Action/Resistance approaches

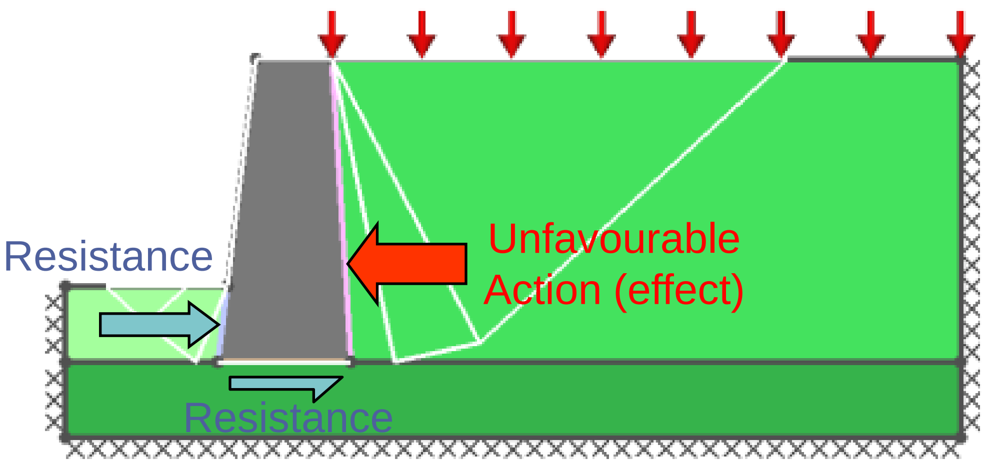
- In Eurocode 7, retaining walls illustrate the concept of an *Action Effect*.



- The action on the wall is a function not only of the applied surface pressure, but the soil self weight ***and*** its strength.

Action/Resistance approaches [2]

Actions and Resistances



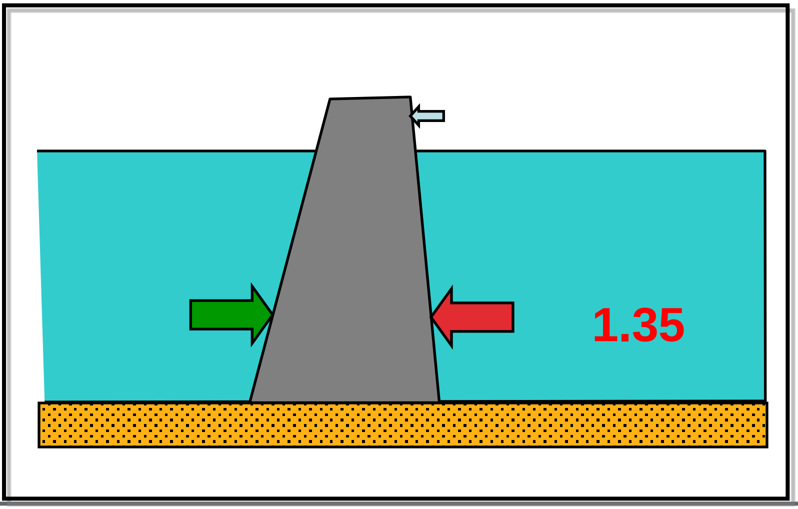
	EC7 DA1/1	EC7 DA2
Permanent unfavourable action	1.35	1.35
Variable unfavourable action	1.5	1.5
Permanent favourable action	1.0	1.0
Resistance	1.0	1.1/1.4

Common source of confusion is to determine at what stage these factors should be applied

Favourable/Unfavourable

2.4.2 Actions

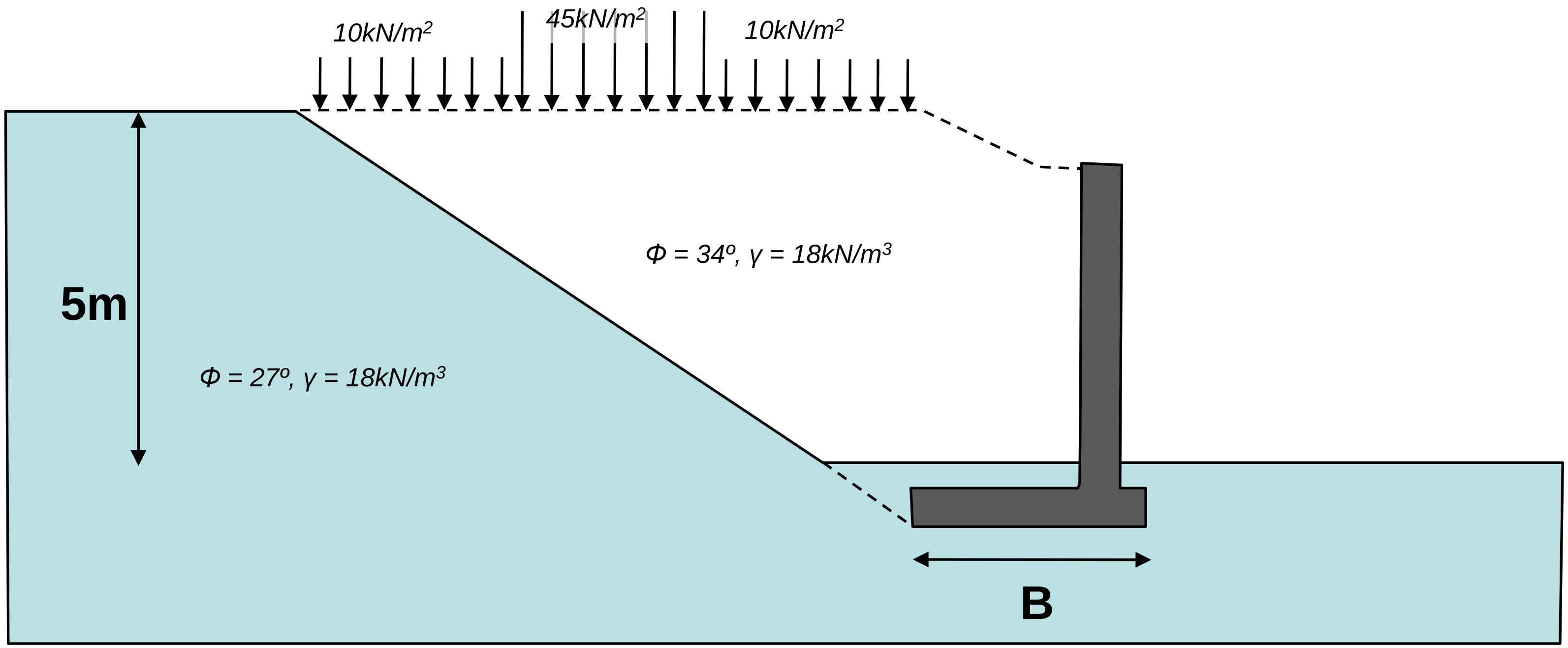
NOTE Unfavourable (or destabilising) and favourable (or stabilising) permanent actions may in some situations be considered as coming from a single source. If they are considered so, a single partial factor may be applied to the sum of these actions or to the sum of their effects.



Retaining Wall Design Example

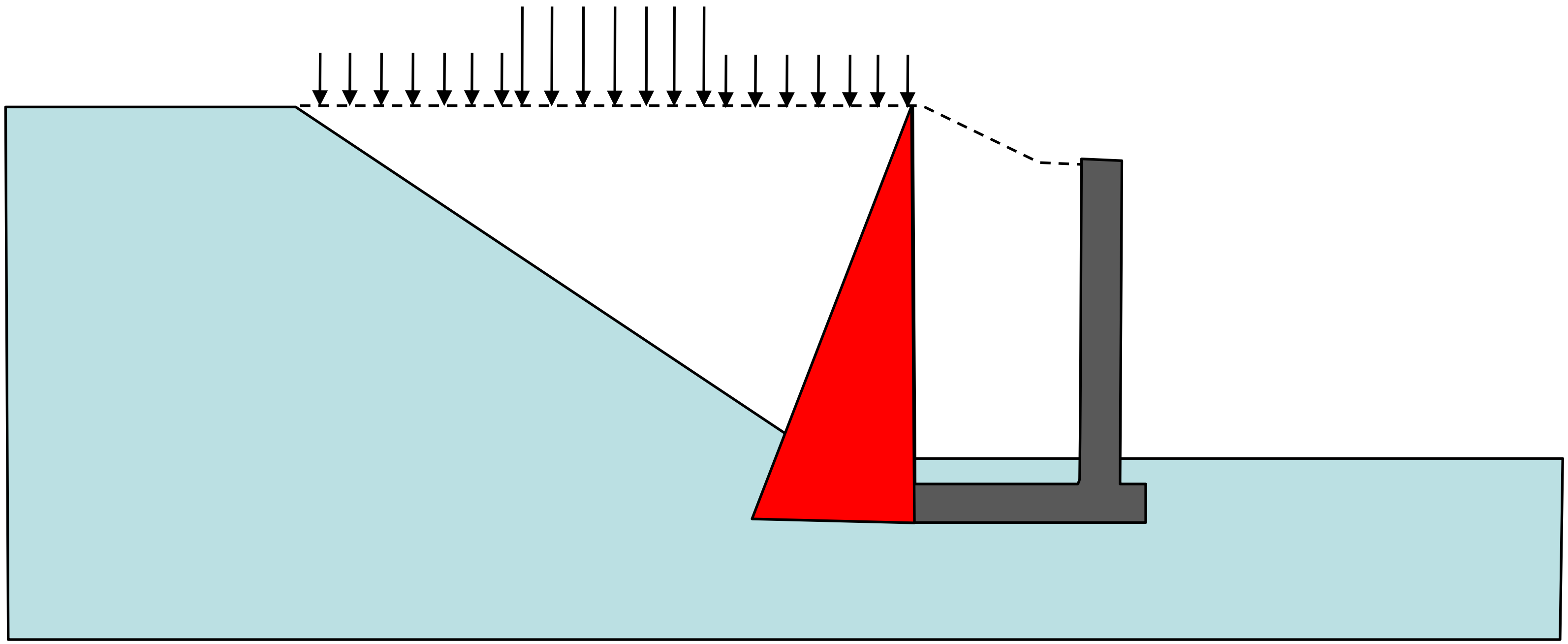
Problem specification

- An existing embankment is to be widened
- A stem wall is to be constructed and backfilled with granular material
- The widened embankment is to take additional loading.
- How wide (B) should the stem wall be?



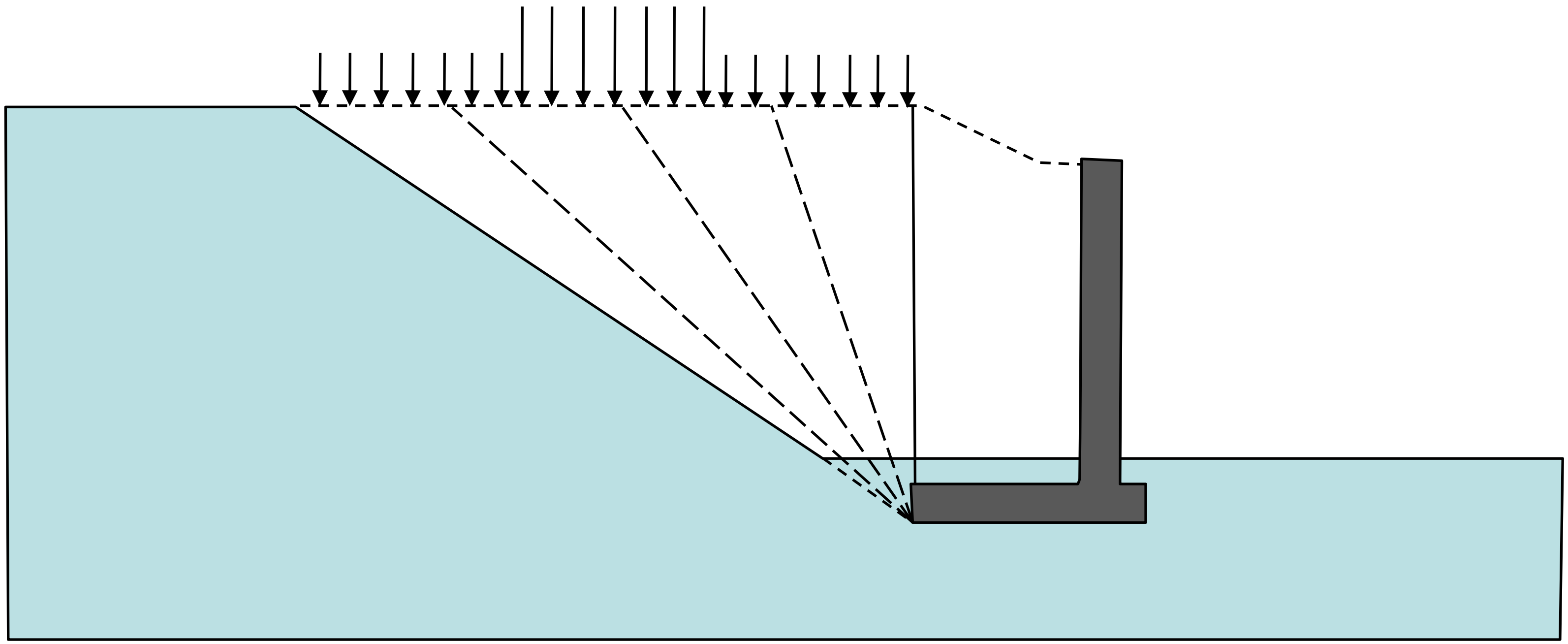
Analysis

Rankine analysis using a virtual back is difficult due to the varied surface loads and the inclined soil interface



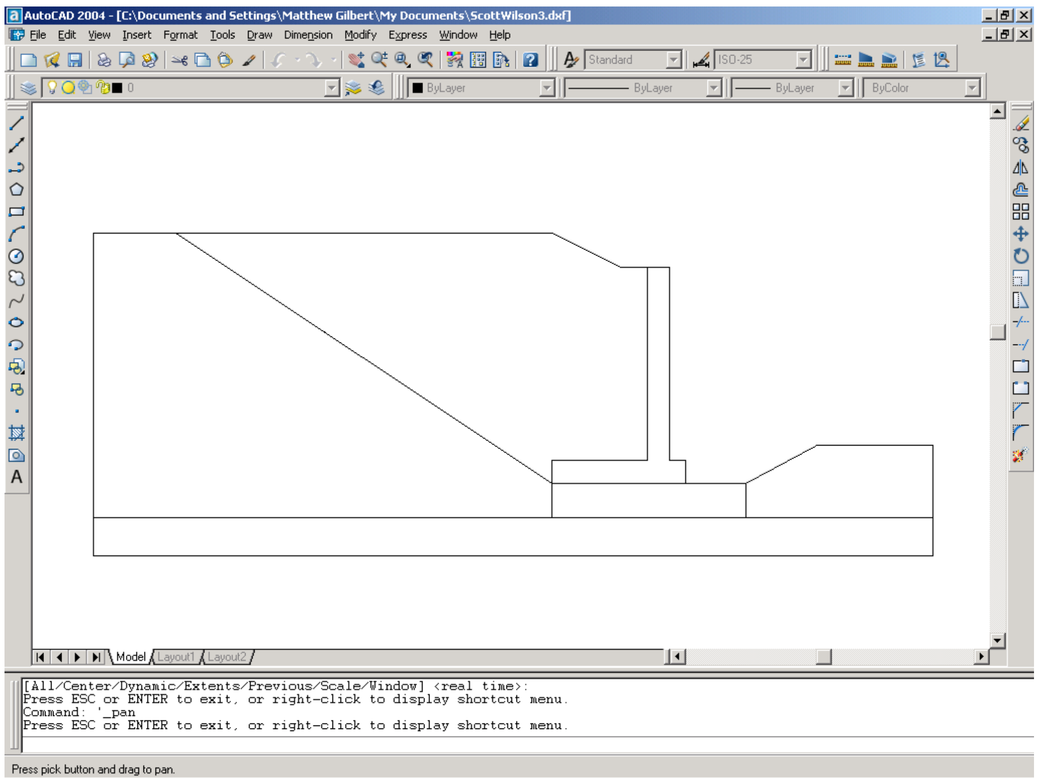
Analysis [2]

Coulomb analysis more appropriate using a virtual back and a range of wedge angles



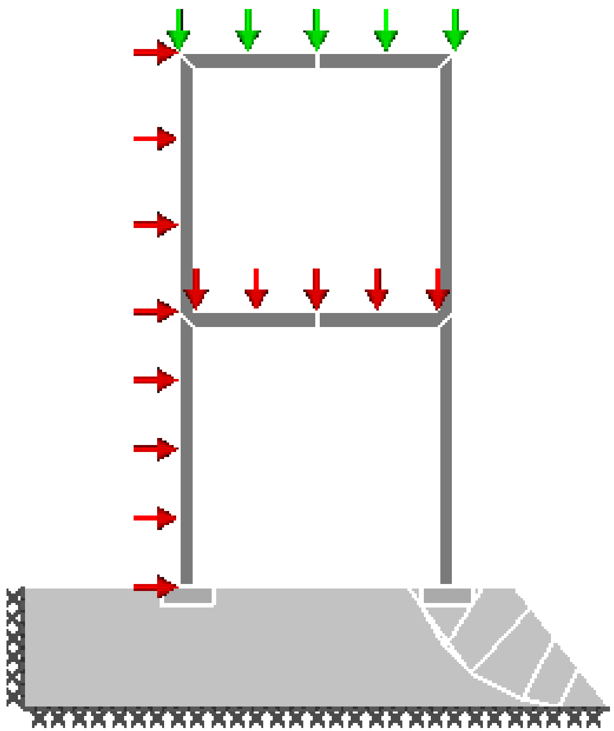
LimitState:GEO Analysis

- Start with a DXF import of initial design



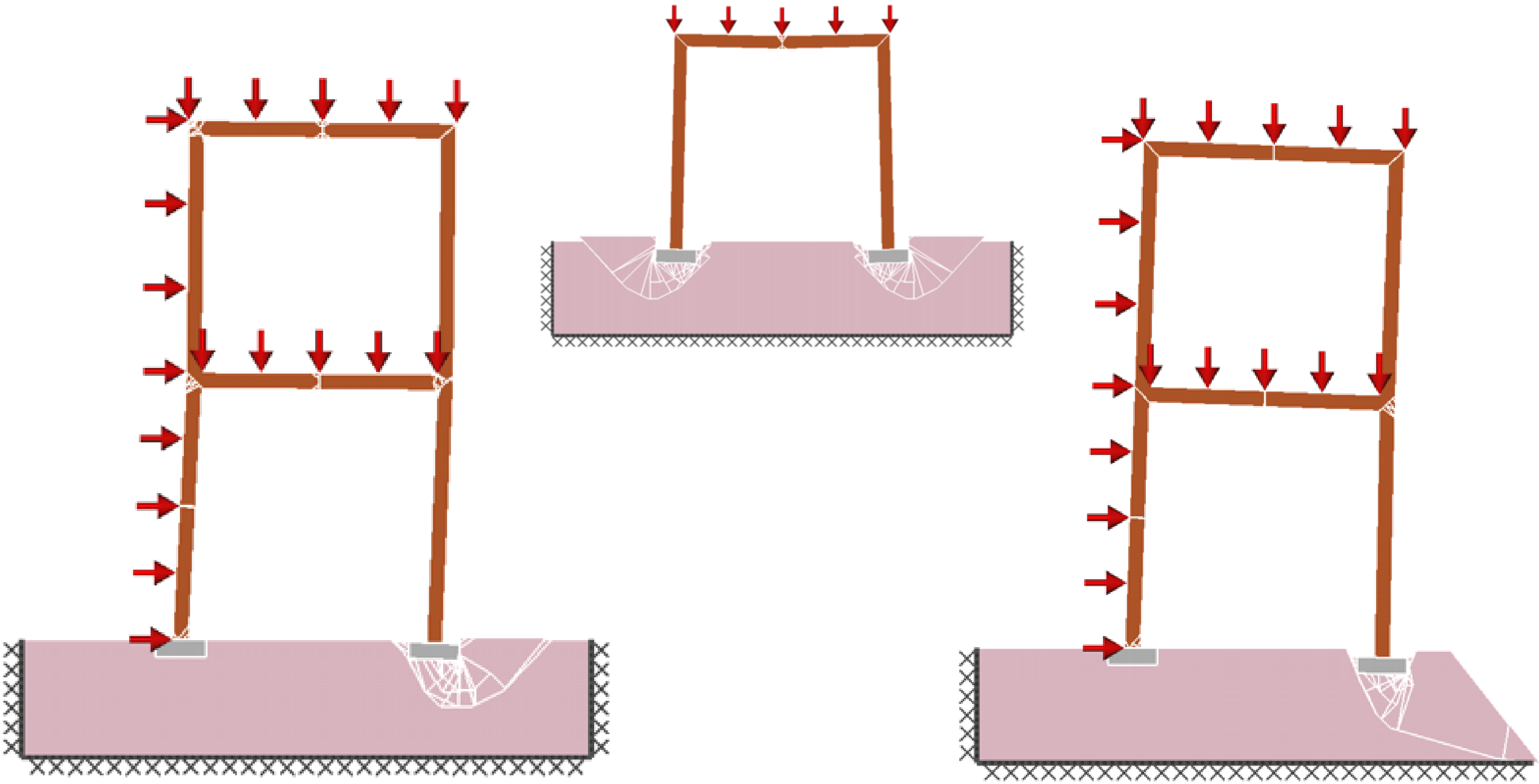
The future

Integrated structural – geotechnical design



Two storey frame on slope

Integrated structural – geotechnical design



Serviceability Limit State

Eurocode 7: “2.4.8 (4) It may be verified that a sufficiently low fraction of the ground strength is mobilised to keep deformations within the required serviceability limits, provided this simplified approach is restricted to design situations where:

- a value of the deformation is not required to check the serviceability limit state;
- established comparable experience exists with similar ground, structures and application method.”

This provides scope for the application of the ‘Mobilized Strength Design’ approach.

Serviceability Limit State [2]

- E.g. BS8002 utilizes a ‘mobilized’ soil strength selected such that **ULS** and **SLS** are simultaneously satisfied.
- The recommended factors of 1.2 on tan ϕ' and 1.5 on c_u are only applicable to soils that are medium dense or firm (or stronger) for **SLS** to be satisfied.
- While the partial factors on soil strength seem similar to those in Eurocode 7 DC2, the concept behind them is completely different.
- The Eurocode 7 factors addresses **ULS** only, but could potentially deal with SLS by ‘borrowing’ the ‘mobilized strength design’ (MSD) approach which is the conceptual basis of BS8002.

Conclusions

Conclusions (Limit State Design – EC7)

- Limit State Design aims to render the probability of a Limit state remote
- It may adopt a Material Factor approach (DA1/2) – this is easy to apply conceptually and to implement numerically
- It may adopt an Action or Action/Resistance Factor approach requiring care in application for more complex problems
- Adopting both approaches can protect against different forms of uncertainty
- EC7 requires distinct separate checks for **ULS** and **SLS**. This contrasts with some existing design codes which simultaneously deal with both **ULS** and **SLS**

Thank you for listening

All analyses shown were run using LimitState:GEO
limitstate.com/geo